On the power-law indenter used in the analysis of nanoindentation unloading curves

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Nanoindentation experiments have become a commonly used technique to investigate mechanical properties of thin films and small volume of materials. Effective indenter shape concept was proposed by Pharr and Bolshakov to explain the nanoindentation unloading curves [1]. From the result of linear elasticity theory and the power-law fit method, they obtain the effective indenter shape, which is described by the following equation:

$$z(r) = 0.548\frac{4(1-\nu^2)pa_{\max}}{\pi E}\left(\frac{r}{a_{\max}}\right)^{2.61}, \quad (0 \le r \le a_{\max}) \tag{1}$$

This power-law indenter, which is derived through the best-fit method (i.e., it is an approximation), is supposed to produce a constant pressure distribution at the peak load [1], (Fig. 10 label). To what degree this indenter can produce a uniform pressure distribution has not been investigated. The widely used Sneddon's solution cannot handle power-law indenters with non-integer powers [2]. In this paper, we use Fu-Chandra solution to obtain the pressure distribution [3, 4]. The result shows that a uniform pressure distribution cannot be obtained with the power-law indenter described by Equation 1.

We consider the indentation of a flat elastic half-space ($z \ge 0$) by a rigid power-law indenter described by Equation 1. The problem is considered in the linear theory of elasticity and the half-space is assumed to be isotropic and homogeneous. The following equations give the relevant displacement and stresses for the half-space. The vertical component of the displacement is denoted by u_z, and the stress components have two subscripts corresponding to the appropriate coordinates. E and ν are Young's modulus and Poisson's ratio of the half-space.

The boundary conditions for the half-space at $z = 0$ are

$$\tau_{zr} = \tau_{z\theta} = 0, \qquad (0 \le r < \infty) \tag{2}$$

$$\sigma_{zz} = 0, \qquad (r > a_{\max}) \tag{3}$$

$$u_z = h - z(r), \qquad (0 \le r \le a_{\max}) \tag{4}$$

where h is the indentation depth. The second term at the right-hand side of Equation 4 describes the indenter shape, and it is given by Equation 1.

The peak load is reached when the contact radius is $a_{\max}$. From the Fu-Chandra solution [3], the indentation depth corresponding to the contact radius $a_{\max}$ is

$$h = \frac{\sqrt{\pi}}{2} \times 3.61\left[0.548\frac{4(1-\nu^2)p}{\pi E a_{\max}^{1.61}}\right]\frac{\Gamma(2.305)}{\Gamma(2.805)} \times a_{\max}^{2.61} \tag{5}$$

Thus,

$$h = 1.552\frac{(1-\nu^2)pa_{\max}}{E} \tag{6}$$

We have the corresponding pressure distribution as

$$\sigma_{zz}|_{z=0} = \frac{E}{2\sqrt{\pi}(1-\nu^2)}\left[-\frac{2h}{\sqrt{\pi\left(a_{\max}^2 - r^2\right)}} + 0.548\frac{4(1-\nu^2)p}{\pi E a_{\max}^{1.61}} \times 3.61\frac{\Gamma(2.305)}{\Gamma(2.805)} \times \Phi(r, 2.61)\right], \quad (0 \le r \le a_{\max}) \tag{7}$$

where

$$\Phi(r, 2.61) = \frac{\sqrt{\pi}}{2} \times 3.61\frac{\Gamma(-1.805)}{\Gamma(-1.305)}r^{1.61} - \frac{a_{\max}^{4.61}}{r^2} \times \left[\frac{1}{\sqrt{a_{\max}^2 - r^2}} - {}_2F_1\left(0.5, -1.805, -0.805; \frac{r^2}{a_{\max}^2}\right)\right]$$

and ${}_2F_1(a, b, c; z)$ is hypergeometric function.

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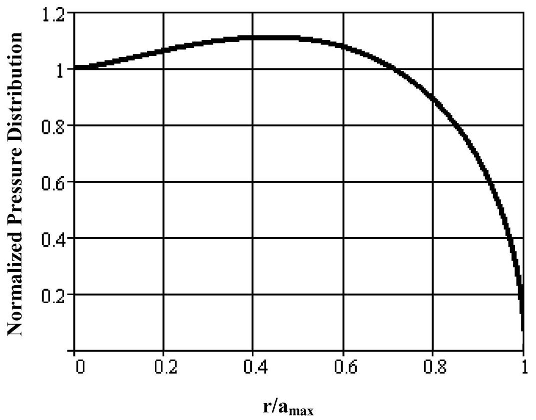

Figure 1 Normalized pressure distribution vs. $r/a_{\max}$. Pressure profile is normalized to the pressure at the indenter center.

Equation 7 is illustrated in Fig. 1, which shows that the pressure distribution is not uniform. The pressure will drop to zero at the edge of the contact area, and the maximum pressure does not happen at the indenter tip.

Thus, in the linear theory of elasticity, the effective indenter given by Pharr and Bolshakov cannot produce a constant pressure distribution.

References

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